23) $\lim_{x\to 0} \frac{\tan 7x}{\sin 3x} \to \frac{0}{0}$

2.61

Key idea:
don't know a
rule with ~~tangent~~, so...
is there a "sin"
lurking
anywhere?

$= \lim_{x\to 0} \left(\frac{\sin 7x}{\cos 7x}\right)\left(\frac{1}{\sin 3x}\right)$

$= \lim_{x\to 0} \left(\frac{7}{3}\right)\left(\frac{\sin 7x}{7x}\right)\left(\frac{1}{\cos 7x}\right)\left(\frac{3x}{\sin 3x}\right)$

$= \frac{7}{3} \quad (1) \quad (1)\left(\frac{1}{1}\right)$

$= 7/3$

$\lim_{x\to 0} \frac{\sin(7x)}{(7x)}$

$7x \to 0$

pattern:

$\lim_{u\to 0} \frac{\sin u}{u} = 1$

$$\lim_{x \to 0} \left(\sin 7x\right)\left(\frac{1}{\cos 7x}\right)\left(\frac{1}{\sin 3x}\right)$$

$0 \cdot \infty$

indeterminate form

26) $\displaystyle\lim_{h\to 0} \frac{\sin h}{1-\cos h}$

$$\lim_{h\to 0} \left(\frac{\sin h}{h}\right)\left(\frac{h}{1-\cos h}\right)$$

Pattern

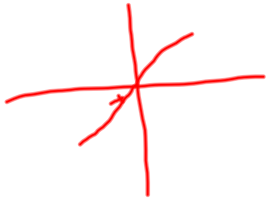
$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

$$1 \qquad \frac{1}{0} \to \infty \qquad \to -\infty \qquad \text{DNE}$$

$$\frac{1}{\frac{1-\cos h}{h}}$$

$$\lim_{h\to 0^+} \frac{h}{1-\cos h} = +\infty$$

$$\lim_{h\to 0^-} \frac{h}{1-\cos h} = -\infty$$



$$\lim_{h\to 0} \left(\frac{\sin h}{1-\cos h}\right)\left(\frac{1+\cos h}{1+\cos h}\right)$$

$$= \lim_{h\to 0} \frac{\sin(h)\,(1+\cos h)}{1-\cos^2 h}$$

$$= \lim_{h\to 0} \frac{\sin(h)\,(1+\cos h)}{\sin^2(h)}$$

$\sin^2 + \cos^2 = 1$

$\sin^2 = 1-\cos^2$

$$= \lim_{h\to 0} \frac{\sin h}{\sin h} \cdot \frac{1+\cos h}{\sin h}$$

$$= \lim_{h\to 0} \frac{1+\cos h}{\sin h} \to \frac{2}{0} \quad +\infty \quad -\infty \quad \text{DNE}$$

$$\lim_{h\to 0^-} \frac{1+\cos h}{\sin h} = -\infty \qquad \lim_{h\to 0^+} \frac{1+\cos h}{\sin h} = +\infty$$

31 27 25

31)

$$\boxed{\sin^2 x+\cos^2 x=1}$$

$$\lim_{h\to 0}\frac{1-\cos 5h}{\cos 7h-1}\rightarrow\lim_{h\to 0}\frac{(1-\cos 5h)}{(\cos 7h-1)}\cdot\frac{(1+\cos 5h)}{(1+\cos 5h)}$$

$$\lim_{h\to 0}\frac{5}{7}\left(\frac{1-\cos 5h}{5h}\right)\left(\frac{7h}{\cos 7h-1}\right)$$

$$=\lim_{h\to 0}\frac{(1-\cos^2 5h)}{(\cos 7h-1)(1+\cos 5h)}\frac{(\cos 7h+1)}{(\cos 7h+1)}$$

$$=\lim_{h\to 0}\frac{(\sin^2 5h)(\cos 7h+1)}{(\cos^2 7h-1)(1+\cos 5h)}$$

$$\lim_{h\to 0}\frac{\sin^2 5h\to 0}{-\sin^2 7h\to 0}\ \underline{\underline{\text{IDK}}}\ \frac{25}{49}\lim_{h\to 0}\left(\frac{\sin^2 5h}{(5h)^2}\right)\left(\frac{(7h)^2}{-\sin^2 7h}\right)\left(\frac{\cos 7h+1}{1+\cos 5h}\right)$$

$$\left(\frac{7h}{-\sin 7h}\right)\left(\frac{7h}{\sin 7h}\right)$$

$$\frac{25}{49}\left(1^2\right)\left(-(1)^2\right)\left(\frac{2}{2}\right)$$

$$=-\frac{25}{49}$$

27) $$\lim_{\theta \to 0} \frac{\theta^2}{1-\cos\theta}$$

$$= \lim_{\theta \to 0} \left(\frac{\theta}{1}\right)\left(\frac{\theta}{1-\cos\theta}\right)$$

$$\frac{\theta}{1} \to 0, \qquad \frac{\theta}{1-\cos\theta} \to \frac{1}{0}$$

$$\Rightarrow \lim_{\theta \to 0} \frac{\theta^2 (1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\theta^2 (1+\cos\theta)}{(1-\cos^2\theta)}$$

$$= \lim_{\theta \to 0} \frac{\theta^2 (1+\cos\theta)}{\sin^2\theta}$$

$$= \lim_{\theta \to 0} \left(\frac{\theta}{\sin\theta}\right)\left(\frac{\theta}{\sin\theta}\right)\left(\frac{1+\cos\theta}{1}\right)$$

$$1 \cdot 1 \cdot 2 = 2$$

25) $\lim_{h\to 0} \frac{h}{\tan h}$

$= \lim_{h\to 0} \frac{h}{\frac{\sin h}{\cos h}}$

$= \lim_{h\to 0} \left(\frac{h \cdot \cos h}{\sin h} \right)$

$= (1)(1) = 1$

28 $\lim_{x\to 0} \frac{x}{\cos\left(\frac{\pi}{2} - x\right)}$

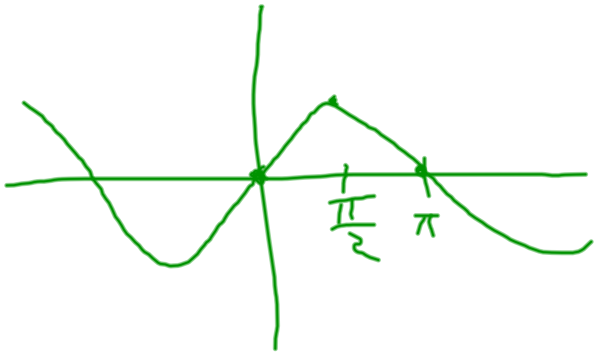
$= \lim_{x\to 0} \frac{x}{\sin x}$

$= 1$

$$\cos\left(\frac{\pi}{2}-x\right) = \sin x$$

$$\cancel{\cos\frac{\pi}{2}\cos(x) +} \sin\frac{\pi}{2}\sin x$$

$$= \sin x$$



$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

29) $\lim_{\theta \to 0} \frac{\theta}{\cos\theta} = \frac{0}{1}$

$= 0$

30 $\lim_{t \to 0} \frac{t^2}{1-\cos^2 t}$

$= \lim_{t \to 0} \frac{t^2}{\sin^2 t}$

$= \lim_{t \to 0} \left(\frac{t}{\sin t}\right)\left(\frac{t}{\sin t}\right)$

$= 1 \cdot 1 = 1$

$\lim_{t \to 0} \frac{t^2}{(1-\cos t)(1+\cos t)}$

$= \lim_{t \to 0} \frac{t}{1-\cos t} \cdot \frac{t}{1+\cos t}$

$\frac{1}{0}$ $\quad \infty \cdot 0 \quad$ 0

32)

$$\lim_{x \to 0^+} \sin\left(\frac{1}{x}\right)$$

does not exist

by reason of oscillation

"think"

$$\sin\left(\lim_{x \to 0^+} \frac{1}{x}\right)$$

$$= \sin\left(\text{"}\infty\text{"}\right)$$

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(20) $\lim\limits_{x\to 0} \dfrac{\sin^2 x}{3x^2} \to \dfrac{0}{0}$ OK

$\lim\limits_{x\to 0} \left(\dfrac{1}{3}\right)\left(\dfrac{\sin x}{x}\right)\left(\dfrac{\sin x}{x}\right)$

$\left(\dfrac{1}{3}\right)(1)(1) = \dfrac{1}{3}$

(16) $\lim\limits_{h\to 0} \dfrac{\sin h}{2h}$

$\lim\limits_{h\to 0} \left(\dfrac{1}{2}\right)\left(\dfrac{\sin h}{h}\right)$

$= \left(\dfrac{1}{2}\right)(1) = \dfrac{1}{2}$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

$\lim\limits_{x\to 0} \dfrac{1-\cos x}{x} = 0$

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$$\lim_{x\to 0^+} \frac{\sin(x)}{5\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$\lim_{x\to 0^+} \left(\frac{\sqrt{x}}{5}\right)\left(\frac{\sin(x)}{x}\right)$$

$$0 \cdot 1 = 0$$

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23)

$$\lim_{x\to 0} \frac{\tan 7x}{\sin 3x}$$

$$\lim_{x\to 0} \frac{\left(\frac{\sin 7x}{\cos 7x}\right)}{\sin 3x}$$

$$= \lim_{x\to 0} \boxed{\frac{7}{3}} \left(\frac{\sin(7x)}{7x}\right)\left(\frac{1}{\cos 7x}\right)\left(\frac{3x}{\sin 3x}\right)$$

$$\left(\frac{7}{3}\right)\left(1\right)\left(1\right)\left(\frac{1}{1}\right)$$

$$= \frac{7}{3}$$

24

$$\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta}$$

$$\lim_{\theta\to 0} \left(\frac{\sin\theta}{\theta}\right)\left(\frac{\sin\theta}{1}\right)$$

$$1 \cdot 0 = 0$$

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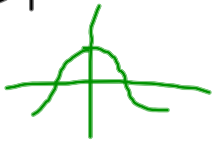
(26) $\lim\limits_{h\to 0} \dfrac{\sin h}{1-\cos h}$

$\boxed{\sin^2 x+\cos^2 x=1}$

$\lim\limits_{h\to 0}\left(\dfrac{\sin h}{h}\right)\left(\dfrac{h}{1-\cos h}\right)$

$1 \qquad \dfrac{1}{0} \quad \begin{matrix} \text{DNE} \\ +\infty \\ -\infty \end{matrix}$



$\lim\limits_{h\to 0^+} \dfrac{h^{+}}{1-\cos h_{+}} = +\infty$

$\lim\limits_{h\to 0^-} \dfrac{h^{-}}{1-\cos h_{+}} = -\infty$

$\lim\limits_{h\to 0}\left(\dfrac{\sin h}{1-\cos h}\right)\cdot\left(\dfrac{1+\cos h}{1+\cos h}\right)$

$=\lim\limits_{h\to 0}\dfrac{(\sin h)(1+\cos h)}{1-\cos^2 h}$

$=\lim\limits_{h\to 0}\dfrac{(\sin h)(1+\cos h)}{\sin^2 h}$

$=\lim\limits_{h\to 0}\left(\dfrac{\sin h}{\sin h}\right)\left(\dfrac{1+\cos h \to 2}{\sin h \to 0}\right)$

$1 \qquad -\infty \quad +\infty \quad \text{DNE}$

$\lim\limits_{h\to 0^+}\dfrac{1+\cos h \to 2}{\sin h \to 0^+} = +\infty$

$\lim\limits_{h\to 0^-}\dfrac{1+\cos h \to 2}{\sin h \to 0^-} = -\infty$

27)

$$\lim_{\theta \to 0} \frac{\theta^2}{1-\cos\theta} = \lim_{\theta \to 0} \frac{\theta^2(1+\cos\theta)}{1-\cos^2\theta}$$

$$\lim_{\theta \to 0} \left(\frac{\theta}{1-\cos\theta}\right)\left(\frac{\theta}{1}\right)$$

$$= \lim_{\theta \to 0} \frac{\theta^2(1+\cos\theta)}{\sin^2\theta}$$

$$= \lim_{\theta \to 0} \left(\frac{\theta}{\sin\theta}\right)\left(\frac{\theta}{\sin\theta}\right)\left(\frac{1+\cos\theta}{1}\right)$$

$$(1)\quad(1)\quad(2) = 2$$

28)
$$\lim_{x\to 0} \frac{x}{\cos\left(\frac{\pi}{2}-x\right)}$$

$$= \lim_{x\to 0} \frac{x}{\sin x} = 1$$

30)
$$\lim_{t\to 0} \frac{t^2}{1-\cos^2 t}$$

$$\lim_{t\to 0} \frac{t^2}{\sin^2 t}$$

$$= \lim_{t\to 0} \left(\frac{t}{\sin t}\right)\left(\frac{t}{\sin t}\right) = (1)(1) = 1$$

$$(1-\cos^2 t)(1+\cos^2 t) = 1-\underline{\cos^4 t}$$

31) $\lim_{h\to 0} \dfrac{1-\cos 5h}{\cos 7h - 1} = \lim_{h\to 0} \dfrac{1-\cos 5h}{\cos 7h - 1}\cdot\dfrac{1+\cos 5h}{1+\cos 5h} = \lim_{h\to 0}\dfrac{1-\cos^2 5h}{(\cos 7h-1)(1+\cos 5h)}$

$\lim_{h\to 0}\left(\dfrac{1-\cos 5h}{5h}\right)\left(\dfrac{7h}{\cos 7h - 1}\right)$

$\to \dfrac{0}{0}$, $\dfrac{1}{0}$

$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

$-\sin^2 = -(1-\cos^2)$

$= -1 + \cos^2$

$\lim_{h\to 0}\dfrac{\sin^2 5h}{(\cos 7h - 1)(1+\cos 5h)}\cdot\dfrac{\cos 7h+1}{\cos 7h+1}$

$= \lim_{h\to 0}\dfrac{(\sin^2 5h)(\cos 7h+1)}{-(\sin^2 7h)(1+\cos 5h)}$

$= \lim_{h\to 0} -\dfrac{25}{49}\left(\dfrac{\sin^2 5h}{(5h)(5h)}\right)\left(\dfrac{(7h)^2}{\sin^2 7h}\right)\left(\dfrac{\cos 7h+1}{1+\cos 5h}\right)$

$\dfrac{-25}{49}\quad (1)^2 \quad (1)^2 \quad \left(\dfrac{2}{2}\right) = \dfrac{-25}{49}$

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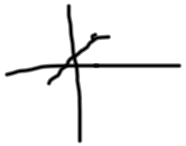
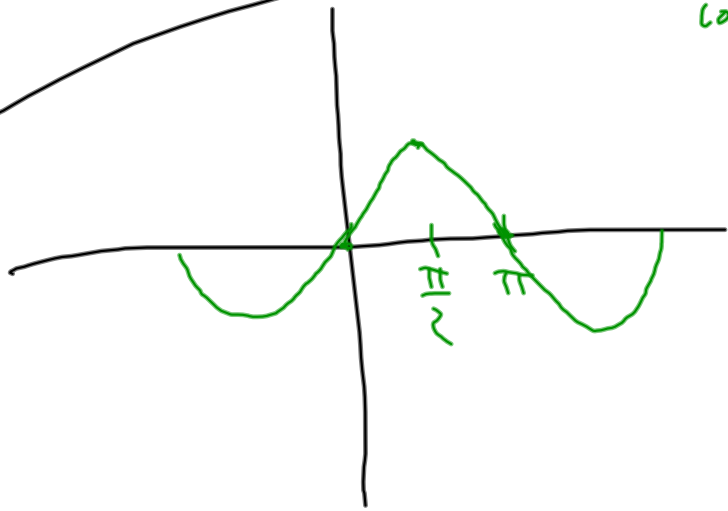
$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$= \cancel{\cos\left(\frac{\pi}{2}\right)\left(\cos(x)\right)} + \sin\left(\frac{\pi}{2}\right)\left(\sin(x)\right)$

$= \sin x$





$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(30^\circ) = $
$15+15$

$\cos^2(15) - \sin^2(15)$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\cos 15 \rightarrow$
$45-30$

$\cos 45 \cos 30 + \sin 45 \sin(30)$

32)

$$\lim_{x \to 0^+} \sin\left(\frac{1}{x}\right)$$

"Think"

$$\sin\left(\lim_{x \to 0^+} \frac{1}{x}\right)$$